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distribute the profits in proportion to the increase in the value of the policy since the date of the last valuation. His argument would point to the former of these methods; but we believe that if that method were applied to an old Office, its injustice would be so obvious that its most strenuous defenders would abandon it.—ED. J. I. A.

Further Considerations on M. Violeine's Solution of a Problem on the Rate of Interest in Loans repayable by Instalments. By PETER GRAY, F.R.A.S., *Honorary Member of the Institute of Actuaries.* (See *ante*, pp. 92 to 99.)

It has been represented to me that, in view of the influential position occupied by M. Violeine, the *quasi* official and authoritative character of his book,* and the currency likely to be thereby imparted to the erroneous principles employed by him in the treatment of problems of the class to which the first of those I have discussed belongs;—I say it has been represented to me that in these circumstances it might be well to be a little more explicit as to his manner of proceeding. In my discussion of the problem in question I did enough, I believe, to shew that M. Violeine's solution is certainly erroneous. I admit however that I did not succeed to my own satisfaction (nor apparently to that of others) in precisely hitting the blot in it, and, as it were, reducing it to its elements. I think that from the consideration I have since given to the matter, I am now able to point out the fallacy by which it is vitiated, and I therefore willingly accede to the suggestion that has been made to me.

The problem may, for our present purpose, be stated as follows:—A person borrows 10,000,000 and grants in return an annuity for forty years, of which the first, second, third, . . . m th payments are 417500, 424680, 431620, . . . $410080 + 7540m - 120m^2$, respectively;† and we are required to determine the rate that he pays for the accommodation. Or, to put it more technically, at what rate does the annuity amortize the loan?

The required rate may be described as either the rate at which the sum borrowed must be improved so as just to suffice for the payment of the annuity; or as the rate at which, at any epoch, the values of the annuity and the sum borrowed are equal.‡ So that, if the beginning of the term be chosen as the epoch of reference, the rate will be that at which the *present value* of the annuity is equal to the sum borrowed; and if the end of the term be chosen, the rate will be that at which the *amount* of the annuity in forty years, and that of the sum borrowed, in the same time, are equal.

* M. V. is described, on the title-page of his book, as "*Membre de l'Ordre impérial de la Légion d'Honneur, chef de Bureau au Ministère des Finances, Auteur de plusieurs ouvrages sur les opérations industrielles*"; and a work of 232 pp., 8vo., now before me, bears the following title:—"Crédit Foncier de France.—Problèmes relatifs aux Intérêts Composés, et plus spécialement aux opérations du Crédit Foncier.—Nouvel emploi des Tables de Violeine pour la solution des Problèmes."

† See pp. 94, 95.

‡ This is in accordance with a general principle which is almost axiomatic; namely, that if two benefits (*certain*, not *contingent*;) be, at a specified rate, of equal value at any one epoch, they will also, *at the same rate*, be of equal value at any other epoch.

It is shown, p. 97, that, referring the values to the former of these epochs—the beginning of the term—at 3·72585 per cent, the present value of the annuity is 9,999,978, as against 10,000,000, the sum borrowed; and if the values be referred to the latter epoch, we shall find that, at the same rate, 3·72585 per cent, the amount of the annuity is 43,199,554, as against 43,199,637, the amount of the sum borrowed in forty years. We conclude therefore that 3·72585 is the rate required, very nearly.

Now the rate assigned by M. Violeine is 3·7613, and we must therefore examine the principle of his solution.

M. Violeine proceeds as follows:—He finds that the amount of the variable annuity at the end of the term, at 3 per cent, is 36,752,392. He then divides this amount by 75·401,260, the amount of a uniform annuity of 1 in forty years, also at 3 per cent. The quotient, 487,424, is obviously the uniform annuity, during the same term, which, at 3 per cent, is equivalent to the given variable annuity. He next finds that at 3·7613 per cent the amounts of the uniform annuity and the sum borrowed, at the end of the term, are equal; in other words, that at this rate the uniform annuity would amortize the loan. And he hence concludes that this is the rate at which the variable annuity amortizes it.

It is here that the fallacy lies. Never surely was inference more unwarranted. It does not in the least follow from the premises. I will put M. Violeine's argument more briefly, thus:—Two benefits (which are not identical), A and B, are equal *at one rate of interest*. One of them, B, *at another rate*, is equal to a third benefit, C. And M. V.'s conclusion is that *therefore, at this rate*, A also is equal to C; from which it would follow of course that *at this rate* A and B are equal.

To make good his inference M. Violeine must be prepared to maintain the following proposition, that if two benefits (which are not identical) are equal at one rate of interest, they are also equal at any or every other rate—a proposition which we know to be absurdly untrue. We therefore altogether reject the rate, 3·7613, assigned by M. Violeine.

I take the present opportunity of giving the formulæ requisite for the general solution of problems of the class to which that under consideration belongs. The enunciation is as follows:—

A loan of S pounds, bearing interest at i per £1, and to be amortized in n years, is represented by k bonds of £ a each, ($ak=S$). The bonds are to be paid off, with a premium of p per £1, as follows:— m the first year, $m+d$ the second, $m+2d$ the third, . . . $m+(n-1)d$ the n th. What is the cost per cent of the loan to the borrower?

It is necessary to observe that m and d are connected by the equation

$$nm + \frac{n(n-1)}{2} d = k,$$

in which n and k are known. So that, one of those quantities being given or assumed, the corresponding value of the other is easily determined.

The two sets of annual payments—the interest, and the amount of the bonds paid off with the premium upon them—form together a variable annuity, to last n years, the rate of interest involved in which, so that its value at any epoch shall equal that of the sum borrowed at the same epoch, and at the same rate, is the rate required. It is convenient to assume the

beginning of the term as the epoch of reference. The required rate is then that at which the present value of the annuity shall be equal to the sum borrowed.

We thus have to satisfy the equation

$$V=S,$$

where V is the present value of the annuity expressed in terms of i , the required rate per £1.

By the general formula for the value of an annuity whose payments are b_1, b_2, b_3 , &c., adapted to this case, we have

$$V = \frac{b_1}{i} + \frac{\Delta^1 b_1}{i^2} + \frac{\Delta^2 b_1}{i^3} - v^n \left(\frac{b_{n+1}}{i} + \frac{\Delta^1 b_{n+1}}{i^2} + \frac{\Delta^2 b_{n+1}}{i^3} \right);$$

and we have to find the value of i which makes this expression equal to S , the sum borrowed.

We easily find for the $(n+1)$ th payment:—

$$b_{n+1} = \{S - an[m + \frac{1}{2}(n-1)d]\}i + a(m + nd)(1+p);$$

differencing twice with respect to n , we get,

$$\begin{aligned}\Delta^1 b_{n+1} &= a[d(1+p) - (m + nd)i], \\ \Delta^2 b_{n+1} &= -adi, \text{ constant.}\end{aligned}$$

And if we make in these $n=0$, we get,

$$\begin{aligned}b_1 &= Si + am(1+p), \\ \Delta^1 b_1 &= a[d(1+p) - mi], \\ \Delta^2 b_1 &= -adi.\end{aligned}$$

These expressions may be verified on M. Violeine's problem; but they are equally applicable to any problem of the same kind. By the use of them the preliminary work on p. 94 is saved.

HOME AND FOREIGN INTELLIGENCE.

COMMERCIAL UNION ASSURANCE COMPANY.

Established 1862.

REPORT TO THE DIRECTORS BY THE ACTUARY, MARCH, 1868.

PART I.

Detailed information upon the Business and constitution of the Office.

(1.) *Business and Regulations.*

The first Policy was granted by the Life Branch on the 1st day of May, 1862.

The number and amount of the several kinds of Policies granted by the Company, of those under which its liability has run-off, and of those remaining in force on the 31st December last (which are the subject of the present valuation), are given in the following Tabular statements.